Kaon and Pion Electromagnetic Form Factor Ratios in the Light-Front

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We have applied the light-front formalism to calculate the electromagnetic form factors for the pion and the kaon from two models at low and high energies in order to explore the differences between such models. We have also compared the results for the ratio $F_K(Q^2)/F_\pi(Q^2)$ with the experimental data up to 10 [GeV/c]^2 and we have observed that the theoretical results are in good concordance for low energies, but they are very different at higher energy scales.

1. Introduction

An interesting laboratory for studying the structure of elementary particles is the electromagnetic form factor $F_\pi(q^2)$ of the pion, as there are a lot of experimental data for this observable [1]. In the case of the K^+ kaon, there is much less knowledge about its electromagnetic form factor $F_K(q^2)$, but it is possible to find some experimental results for $q^2 > 1$ [GeV/c]^2 [2]. Further, the recent high-statistics Brookhaven experiment E865 [3] has provided us with more information on F_K. The amplitude for the decay $K^+ \to \pi^+ e^+ e^-$ was measured for q^2 up to 0.125 [GeV/c]^2 (the maximum value for the kinematics of this kaon decay), that allows to obtain indirectly the value of $F_K(q^2)$ [4].

The main objective of the light-front models used in this work is to describe consistently hadronic bound state systems for high and low Q^2 regimes. In light-front models, the bound state wave function is defined on the hypersuperface $x^+ = x^0 + x^3 = 0$ and it is covariant under kinematical boosts, due to the stability of Fock-state decomposition under such boosts [5].

As described in Ref. [6,7], problems related to the rotational symmetry breaking make the results of electromagnetic form-factor calculations in the light-front formalism dependent on which component of the electromagnetic current is used to obtain the form-factors.

At low momentum transfers, the non-perturbative regime of QCD is more important when compared with high momentum transfers dominated by the perturbative regime of QCD. Perturbative QCD works well after 1.0 [GeV/c]^2 and dominates above 5.0 [GeV/c]^2.

The studies on light-flavor vector and scalar mesons are important because they indicate a direction to understand why QCD works in the non-perturbative regime and, also, why the pseudoscalar mesons are the observed light hadrons related with chiral symmetry breaking.

As known for spin-1 particles [6], the plus component ("J^+") of the electromagnetic current is not free from pair term contributions in the Breit frame ($q^+ = 0$), so the rotational symmetry is broken if they are dropped out. Thus, the matrix elements of the electromagnetic current in the light-front formalism have the valence contribution to the electromagnetic current but also other contributions should be considered. That contribution corresponds to pair terms added to the matrix elements of the electromagnetic current [6].

In the present work, one type of the vertex function is used in order to calculate the pion electromagnetic form-factor for the $\pi - q\bar{q}$ ver-

tex. The light-front models for the pion and the kaon which were presented at previous works [6] are applied to high momentum transfers.

In section II, we present the formalism for the decay constants and electromagnetic form factor of the pseudoscalar mesons for a nonsymmetric vertex model. In our calculations, we also use a symmetric vertex model of the pion and kaon given in this section. In the last section, we show our numerical results for the ratio between the kaon and pion electromagnetic form factors. We also discuss the main relevant points of the results.

2. Electromagnetic Form Factor for Pseuoscalar Mesons

The electromagnetic form factor for pseudoscalar particles can be obtained from a covariant expression as:

$$< p|J^{\mu}|p' >= (p'+p)^{\mu} F_{PS}(q^2), \tag{1}$$

where J^{μ} is the electromagnetic current and $F_{PS}(q^2)$ is the pseudoscalar electromagnetic form factor.

The pseudoscalar pion decay constant is given by

$$ip^{\mu} f_{PS} = \frac{m}{f_{PS}} N_c \int \frac{dk^4}{(2\pi^4)} Tr[\mathcal{O}] \Lambda_M(k,p), \tag{2}$$

where

$$\mathcal{O} = \gamma^{\mu}\gamma^5 S(k)\gamma^5 S(k-p).$$

2.1. Pion Form Factor

The pion electromagnetic current can be written as:

$$J^{\mu} = e(p^{\mu} + p'^{\mu}) F_{\pi}(q^2). \tag{3}$$

In the equation above, in general, it is possible to extract the form factor using either the plus or minus components of the electromagnetic current, J^+ and J^-, respectively. In a reference frame, where the plus component of the moment transfer $q^+ = q^0 + q^3$ is nonzero, the electromagnetic form factor has two contributions:

$$F_{\pi}(q^2) = F_{\pi}^{(I)}(q^2) + F_{\pi}^{(II)}(q^2), \tag{4}$$

where $F_{\pi}^{(I)}(q^2)$ and $F_{\pi}^{(II)}(q^2)$ are the valence and the non-valence terms, respectively.

Table 1
Parameters for the nonsymmetric vertex model (NSM) and the light-front covariant model (LFCM). The scale parameters λ_π and λ_K are fitted to the corresponding decay constants.

NSM	
m_u	0.220 GeV
$m_{\bar{s}}$	0.419 GeV
m_R	0.946 GeV
f_π	0.101 GeV
m_π	0.140 GeV
m_{K^+}	0.494 GeV
$\langle r_{\pi^+} \rangle$	$0.67 fm$
LFCM ($n=2$)	
m_u	0.220 GeV
m_{π^+}	0.140 GeV
λ_π	0.542 GeV
$\langle r_{\pi^+} \rangle$	0.576 fm
f_{π^+}	0.0924 GeV
$m_{\bar{s}}$	0.344 GeV
m_{K^+}	0.454 GeV
λ_K	0.621 GeV
$\langle r_{K^+} \rangle$	0.513 fm
f_{K^+}	0.113 GeV

From the analytical integration of the k^- loop momentum in Eq. (4), one finds two possible intervals of k^+ which give nonzero contribution: (i) $0 < k^+ < p^+$ and (ii) $p^+ < k^+ < p'^+$. The interval (i) corresponds to the valence contribution to the electromagnetic current and the second interval (ii) is the non-valence contribution. The sum of both contributions gives the covariant form factor, which depends only on q^2 and not on the particular value of q^+.

The electromagnetic form factor $F_{\pi}^{(NSY)}(q^2)$ for the pion can be expressed with the nonsymmetric vertex model (see the reference [7] for details) as:

$$F_{\pi}(q^2) = \imath e \frac{m^2 N^2}{p^+ f_\pi^2} N_c \int \frac{d^2 k_\perp dk^+}{2(2\pi)^4} I(k_\perp, k^+), \tag{5}$$

where

$$I = \frac{M\Theta}{k^+(p^+ - k^+)^2(p'^+ - k^+)^2}$$

with M given by:

$$-4\bar{k}^- k^-(k^+ - p^+)^2 + 4(k_\perp^2 + m^2)(k^+ - 2p^+) + k^+ q^2,$$

$$\Theta = \frac{\theta(k^+)\theta(p^+ - k^+)}{H(k_\perp, k^+)},\ \bar{k}^- = \frac{f_1 - \imath\epsilon}{k^+},$$

and

$$H = (p^- - \bar{k}^- - \frac{f_2 - \imath\epsilon}{p^+ - k^+})(p'^- - \bar{k}^- - \frac{f_3 - \imath\epsilon}{p'^+ - k^+}) \times$$
$$(p^- - \bar{k}^- - \frac{f_4 - \imath\epsilon}{p^+ - k^+})(p'^- - \bar{k}^- - \frac{f_5 - \imath\epsilon}{p'^+ - k^+}).$$

The function f_i for $1 \leq i \leq 5$ is defined as $r_\perp^2 + a^2$, with $(r, a) = (k, m); (p - k, m); (p' - k, m_R); (p - k, m); (p' - k, m_R)$, respectively to the sequence 1 to 5. The constituent quark mass is m and the regulator mass is m_R given in Table I.

2.2. Kaon Form Factor

The expression for the kaon electromagnetic form factor is given by:

$$< p|J^\mu|p' >= e(p'^\mu + p^\mu)F_K(q^2), \qquad (6)$$

where J^μ is the electromagnetic current and $F_K(q^2)$ is the kaon form factor.

The electromagnetic form factor of the kaon

$$F_K(q^2) = F_q(q^2) + F_{\bar{q}}(q^2)\ , \qquad (7)$$

receives contribution form the quark and anti-quark currents. With the nonsymmetric vertex [9] in the light-front coordinates the contribution of the quark current to the form factor can be expressed as:

$$F_q(q^2) = 2\imath e_q \frac{N^2 g^2 N_c}{2p^+} \int \frac{d^2k_\perp dk^+ dk^-}{2\ (2\pi)^4}\ \ I \qquad (8)$$

where $I = I_1(k_\perp, k^+, k^-) \times I_2(k_\perp, k^+, k^-)$ and the functions I_1 and I_2 are given by:

$$I_1 = \frac{A(k_\perp, k^+, k^-)}{k^+(p^+ - k^+)^2(p'^+ - k^+)^2(k^- - \frac{f_6 - \imath\epsilon}{k^+})}$$

where $f_6 = k_\perp^2 + m_{\bar{q}}^2$ and

$$I_2 = \frac{-4k^+ m_q^2 + 8(k^+ - p^+)m_q m_{\bar{q}}}{B(k_\perp, k^+, k^-)},$$

with

$$A = -4(k^- k^{+2} - k^+ k_\perp^2 - 2k^- k^+ p^+$$
$$+ 2k_\perp p^+ - \frac{k^+ q}{4} + k^- p^+).$$

and

$$B = (p^- - k^- - \frac{f_2 - \imath\epsilon}{p^+ - k^+})(p'^- - k^- - \frac{f_3 - \imath\epsilon}{p'^+ - k^+}) \times$$
$$(p^- - k^- - \frac{f_4 - \imath\epsilon}{p^+ - k^+})(p'^- - k^- - \frac{f_5 - \imath\epsilon}{p'^+ - k^+}).$$

The functions f_2 to f_5 in the equation above are defined as previously replacing m by m_q.

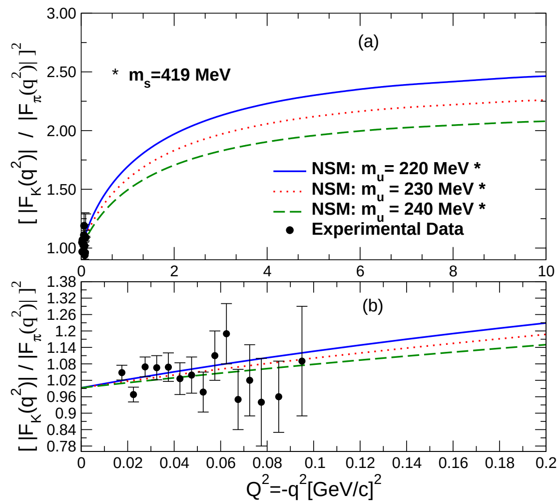

Figure 1. (a) Ratio between the kaon and pion form factors for the nonsymmetric model for Q^2 up to 10 $[GeV/c]^2$; (b) calculations for Q^2 up to 0.2 $[GeV/c]^2$ compared to the experimental data [1].

2.3. Light-Front Covariant Model: The Bethe Salpeter Amplitude

The Bethe-Salpeter amplitude is calculated with the light-front covariant model [8]:

$$\Lambda_M\,(k, p) = \frac{(k_1^2 - m_1^2)\Gamma_M(k_2^2 - m_2^2)}{(k_1^2 - \lambda_M^2 + i\epsilon)^n(k_2^2 - \lambda_M^2 + i\epsilon)^n}, \qquad (9)$$

where $k_1 = k$ and $k_2 = p - k$. λ_M is the scale associated with the meson light-front valence wave function, n is the power of the regulator and m_1 and m_2 are the quark masses within the meson bound state.

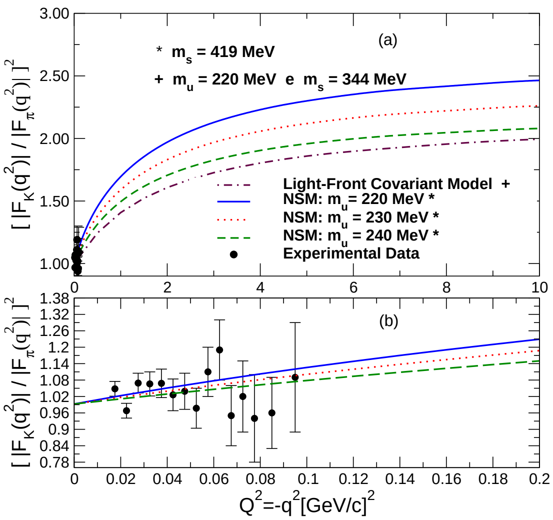

Figure 2. (a) Ratio between the kaon and pion form factors for the Light-Front Covariant Model up to Q^2= 10 $[GeV/c]^2$; (b) calculations for Q^2 up to 0.2 $[GeV/c]^2$ compared to the experimental data [1].

3. Results and Conclusions

The parameters of our models, NSM and LFCM, are given in the Table I. In figure 1, we show the numerical results of the ratio between the kaon and pion electromagnetic form factors obtained with NSM compared to the experimental data. In figure 1(a) (upper frame), we show our results for Q^2 up to 10 $[GeV/c]^2$. In order to show in detail the comparison with available experimental data, it is presented the ratio for Q^2 up to 0.2 $[GeV/c]^2$, in figure 2(b)(lower frame).

We show in figure 2(a) calculations for the ratio using the meson Bethe-Salpeter amplitude defined with the vertex given by LFCM. We compare that to the calculations with NSM up to 10 $[GeV/c]^2$. From the figure, it is clear the sensitivity of the ratio to the different models. From figure 2(b), we realize that both models can describe the experimental data up to Q^2= 0.2 $[GeV/c]^2$. We conclude by observing both frames in figure 2 that the two models are in good agreement with the experimental data for low momentum and show significant dependence on the quark mass for high momentum transfers, which can be clearly seen for the NSM. This is possibly correlated to modification of the pion and kaon decay constant when changing the constituent quark mass, which appears at high momentum transfers as the wave functions of the pion and kaon at short distances are essentially defined by f_π and f_K, respectively.

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